Co-operative Learning in Mathematics
-
Java WebMatrix
An attempt to foster asynchronous co-operation via email in Higher Education

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Abstract

Different reasons lead to an increase in using computers to facilitate human-human interaction. This trend has already been identified in the early 90's by Ellis, Gibbs, and Rein (1991). As the author recently read a collection of a private email group of students at the university of Karlsruhe, Germany he identified some barriers introduced by Information and Communications Technology (ICT). One problem was the illustration of abstract ideas in a concrete form on a computer. The problem occurred when the students tried to display and modify mathematical notation on a computer in order to co-operate via email. In contradiction to Ellis et al., cited above, this results in a large barrier to use computers for human-human interaction, especially in co-operative learning. However, since Vygotsky and his famous zone of proximal development (Vygotsky, 1986) the benefits deriving from learning in groups are known.

Hence the leading idea of this paper is to foster and facilitate co-operative learning via computers. This paper does not compare face-to-face co-operative learning with co-operative learning with computers and email. Rather it tries to alleviate co-operative learning in situations where face-to-face co-operation is not possible, which might happen out of class or in self-directed learning without a teacher. One part is to overcome the barrier described above, which means to facilitate the illustration of abstract ideas in a concrete form on a computer. To go into detail, a tool has to be developed to display and modify mathematical notation on a computer. It is known to the author that there are existing tools to do that, like Maple or LaTeX. But these current available system are far too complex to use for non-experts in the field and need a long time to get used to. This, however, hampers learners like students in the first years of university to use computers for co-operation.

This paper reports a case study which has been attempted in order to analyze how such a tool for mathematical notation becomes used and if it fosters and facilitates co-operation online. The tool is called Java WebMatrix and was specially developed for this case study. It's function is limited to display and modify matrices on a computer and generate text output for easy exchange of information via email. In addition to this different kinds of literature have been reviewed in order to set the environment for the case study and the design of the case study itself. Amongst others literature about creating mathematical learning, computer based learning, case studies and co-operative learning are analyzed.

Contents

1 Introduction

This introduction tries to provide a basic understanding of what matrices are and why they are important. It is clear to the author that the grasp of this is important to understand the idea and motivation for this paper. Hence the first subsection attempts to explain what a matrix is whereas the next subsection focuses more on the purpose of matrices. After this an outline of the rest of the document will be given before a short summary itemizes the most important parts of this introduction again.

At this point the author wants to mention that the whole document including it's different kinds of mathematical notation is created using LaTeX, a powerful, but complex tool. The aim of Java WebMatrix (JVM) is to provide a similar functionality for talking about matrices, but it should be much easier to use.

1.1 Definition and basic operations of a matrix

It is evident to the author how important it is for the reader of this paper to understand what matrices are. Without knowing what matrices are or why they are used this whole paper makes no sense. However, despite a matrix is a well-defined construct for a mathematician, it is very hard to explain them and prove their importance to readers who are not already familiar with them. First, this subsection makes the attempt to define and explain what a matrix is. The next subsection then highlights the importance and the use of matrices. If you are used to matrices you might skip immediately to the Literature Review, section 2.

Further down in this subsection a mathematic correct definition of a matrix will be given. Here an attempt of the author to explain them in a more intuitive way. Probably every reader has solved mathematic problems with notations looking like the following

$$2p + 3t = 10$$

$$4p + 9t = 23$$

You can solve this problem and come to the solution that $p = 3,5$ and $t = 1$, and this are maybe the amounts of apples Paul and Tim are eating in an average week. Unfortunately, in reality there are quite often much more variables. Therefore you can leave away the name of the variables and the operations, which will look like the following

$$\begin{pmatrix} 2 & 3 & 10 \\ 4 & 9 & 23 \end{pmatrix}$$

This is already a simple example of a matrix. It is just an efficient way to display and modify a problem mathematically. And the big advantage is that there are certain modifications to a matrix allowed without changing the represented problem. Most important there are three basic operations on matrices :

1. Interchanging two rows or columns

2. Adding a multiple of one row or column to another

3. Multiplying any row or column by a nonzero element

Therefore we can use the second basic operation at our matrix and multiply row 1 with -2 and add it to row 2.

$$\begin{pmatrix} 2 & 3 & 10 \\ 4 & 9 & 23 \end{pmatrix} \begin{array}{l} {\scriptstyle -2} \\ {\scriptstyle +} \end{array}$$

This would result in the following matrix

$$\begin{pmatrix} 2 & 3 & 10 \\ 0 & 3 & 3 \end{pmatrix}$$

If you retranslate this matrix back in the starting representation you end up with

$$2p + 3t = 10$$

$$0p + 3t = 3$$

and can immediately extract the solution. Obviously, in this example it made little sense first to abstract the problem into matrix form, solve it and then retranslate it. However, the more complex the problem gets the more useful the matrix representation becomes.

After this as easy as possible introduction now the common definition of a matrix mathematicians would use and another example how it looks if you use them to solve a problem. Below how a mathematician would notate a matrix if he/she wants to explain what a matrix is.

$$\begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,m} \\ a_{2,1} & a_{2,2} & \dots & a_{2,m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,m} \end{pmatrix}$$

To have the maximal flexibility an "a" with two indices is used. The first one indicates the row, the second one the column position. Therefore this matrix has n rows, m columns, and m x n elements.

To prevent from getting completely lost in abstraction the author doesn't show the basic operations on the above general representation of a matrix. Rather a fixed 3 row 3 column matrix is used to show another, a little bit more complex example how the basic operations on a matrix can facilitate an underlying problem. The example below is taken from Roehricht and Kauers.

$$
\begin{pmatrix} 1 & 1 & 1 \\ t & 2t & 2 \\ t+1 & 0 & 2t \end{pmatrix}
\begin{array}{l} \;\; -t \quad -(t+1) \\ + \\ \qquad\quad + \end{array}
\rightsquigarrow
\begin{pmatrix} 1 & 1 & 1 \\ 0 & t & 2-t \\ 0 & -t-1 & t-1 \end{pmatrix}
\begin{array}{l} \\ \\ + \mid \cdot (-1) \end{array}
$$

$$
\rightsquigarrow
\begin{pmatrix} 1 & 1 & 1 \\ 0 & t & 2-t \\ 0 & 1 & -1 \end{pmatrix}
\begin{array}{l} \\ \quad -t \\ \quad + \quad \mid : 2 \end{array}
$$

$$
\rightsquigarrow
\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}.
$$

To somebody who is used to matrices and their operations this visualization is very clear and well-defined. The next subsection will try to explain the purpose and the importance of matrices.

1.2 Purpose of matrices

This subsection tries to highlight the importance and purpose of matrices. If you have read through the previous subsection you already know that matrices are a mathematical tool to work with problems. Their purpose is obviously to have an abstracted illustration of a problem. The nice attribute of this representation is that there are standard ways how to work with matrices. Therefore it is possible to solve a problem in a standardized way. But in contrast to tools taught in school mathematics, matrices are well suited to work with realistic, real life problems. For example they are the common tool to describe three dimensional experiments in physics. But a matrix is not only a very useful tool in physics, they are also widely used in business, statistics and computer science environments. The next subsection will now give a broader overview over the rest of the document.

1.3 Roadmap

After the summary which finishes the introduction a literature review, section 2, will give a short overview of the theories which helped significantly to design the case study. Amongst other things literature about co-operative learning, mathematical teaching and learning, computer-based learning and case studies are analyzed in respect of their meaning for this paper. After this the actual design and implementation of the case study is described and justified in reference to the literature review in section 3. In the same section the conducted case study is evaluated. The last section discusses the outcomes of the preceding sections and demonstrates which parts might be interesting for further investigations. As already said the summary of the introduction follows.

1.4 Summary

After explaining what matrices are this introduction has shown that matrices are a mathematical tool which can represent different underlying problems. The nice aspect is that there are standardized ways to work with matrices and therefore standardized ways to facilitate the represented problems. Hence matrices are a relevant tool in many different areas of science. In addition to this the introduction has given a short outline of the rest of the document. In the following literature review the mentioned parts of literature are analyzed looking at their meaning for the case study about the JVM.

2 Literature Review

This literature review reveals principles in literature for the design of the JVM and it's environment. First, the sense and purpose of group learning activities is highlighted. In the same subsection the author justifies his decision to choose co-operative learning as underlying methodology. The next two subsections then analyze current advises for the design of mathematical learning and learning with computers. In the last but one section directions about how to design case studies are investigated, as this paper is based on a case study. At the end a short summary repeats the main principles discovered.

2.1 Why co-operative learning?

A justification for group activities in general is seen in the work of Vygotsky. According to his theories "individual consciousness is built from outside through relations with others"(1986, p. xxiv). One expression he coined was that of a psychological tool, which is for example transforming the natural human abilities and skills into language or mnemonic techniques. These psychological tools are necessary as they construct the higher function thinking skills together with interpersonal relations (Vygotsky, 1986). The need of the development of these thinking skills is stated in the much more recent work of Wegerif (2002), as these skills are relatively generally accepted "learning strategies that can be drawn out of some contexts and applied again in new contexts" (p. 2). Another popular expression created by Vygotsky is the zone of proximal development (zo-ped), which is defined by the range of a student's ability on a given task depending on the fact if their is assistance of a more capable peer or not. According to this theory a student is capable to solve more complex problems with assistance of a more capable peer than without.

After analyzing positive aspects of group activities in general, the aspects of co-operative learning in particular are investigated. To clarify the special attributes of co-operative learning, collaborative learning is also described to highlight the differences. Each of these two major directions in group learning includes several sub-directions which are not discussed in this paper. Collaborative learning is about high level thinking processes, complex techniques and includes discussion (Gillies & Ashman, 2003) In opposition to this co-operative learning is more about limited and well defined tasks and is useful for basic skill acquisition. Matrix displaying and operations can include all of the attributes of collaborative learning. Nevertheless the author of this paper decided to use co-operative learning as guidelines for the group environment described in the next section. This decision was made as the experiment investigated in this paper has only a duration of two weeks. Successful collaborative group work however should include pre-acquisition of social skills and strategies to learn in groups (Gillies & Ashman, 2003). Another argument against collaborative learning is that there should be assessment of the individual in the group. Disregarding this guideline would lead to less working individuals, according to the theory of social loafing (Gillies & Ashman, 2003). From this point of view the author favors again co-operative learning, as it is easier to individually access basic steps done by individuals than to analyze the processes of higher order thinking they have gone through. The author does not want to say that co-operative learning is better at working with matrices, but it seems to be more suitable in the framework this paper is done.

2.2 Concepts of mathematical teaching and learning

This subsection is guided by ideas and guidelines in *Beyond Constructivism* (Lesh & Doerr, 2003). For more details look at the reference itself, here the author only summarizes the key concepts useful for the design of the case study. As a major change in mathematic education Lesh and Doerr suggest that the traditional definition of problem solving teaching should be changed. In the traditional way students have to solve simplified problems without meaning. It is hard to solve the more complex realistic problems. In the suggestion made in *Beyond Constructivism* students should be taught to solve model-eliciting problems. This problems are much more similar to real life problems in which mathematics is useful. The key idea is that the meaning in the task facilitates the complex aspects. In addition these model-eliciting problems are described as thought-revealing, which means that the teachers can assess the thinking processes the students have gone through. This is very important, as "thinking mathematically is about constructing, describing and explaining" (Lesh and Doerr, 2003, p. 16), not only about completing standardized tasks.

Another key idea in *Beyond Constructivism* is the idea of representing the complexity of problems in different media to contain all information. Lesh and Doerr are referring to this construct of thinking as a model, which is in its own definition a conceptual system that is expressed using external notation symbols. The main problem of these models or conceptual systems is summed up in the following quote

> "Conceptual systems are similar to icebergs in the sense that a large portion of what is important is not visible in any single media"(Lesh and Doerr ,2003, p. 13)

In addition to the problem the first statement points out, they see a further problem as "different media emphasize different aspects of the system ". The solution to these problems is given by using different media, for example entities like graphs, writing, language, tables and mathematical notation(eg a matrix).

2.3 Computers-based learning regarding mathematics

Following needs which have been identified in learning environments which heavily rely on the use of computers are given in a nutshell, especially considering computer-based co-operation. Oliver and Omari (2001) have identified missing feedback loops as a major problem in earlier implementations of co-operative learning using a computer. They concluded their own experiment with 240 first year students in a multimedia university course in Australia with expressing the need for more structure and guidance in future experiments. Also they see a need for a deliberate strategy to help students to reflect about their solutions, informing and encouraging feedback might be able to take this function. On the technical side Oliver and Omari point out that it is very important that the used technology is reliable and useable at all times. As mentioned in the subsection before, it is very important to use a variety of different media. Lesh and Doerr are using the term eMedia to refer to illustrations on a computer, and for each traditional media there is a corresponding eMedia. The importance and strengths of these eMedia is stated in the following quote

> "But, whereas traditional tend to emphasize static objects, relationships and events, their corresponding eMedia are easier to manipulate; they tend to be more dynamic (focusing on actions and transformations); they are interactive (by responding to actions on them), and they are linked (so that actions carried out in one medium are reflected automatically in other media)" Kaput in *Beyond Constructivism*, p. 267 (Lesh & Doerr, 2003)

Together with the principle to better build small-but-easy-to-extend rather than large-and-difficult-to-reduce computer tools(Lesh & Doerr, 2003), also referred to as plug-in software components (Grundy & Hosking, 2002) this subsection has provided a number of guidelines and principles how the environment in which the JVM is used should be designed.

2.4 Design of case studies

Yin' states that a "case study is a comprehensive research strategy including design and data collection"(Yin,2003, p. 13) that links the data and the conclusions to the initial question of study. This subsection first shows the situations a case study is an appropriate research strategy and then highlights the main guidelines which Yin gives for the design of case studies (Yin, 2003). In particular, case studies are well suited to answer 'How' and 'Why' research questions. There are five components which should be regarded for the design of a case study. First, the study question is very important. Second, it's propositions have to be clear before conducting the case study, as they are helping to decide where to look for the desired information. Third, the unit(s) of analysis, which could be an individual, an organization or interpersonal communication and fourth, the logic linking the data to the proposition have to be defined. And last the criteria for interpreting the findings has to be stated. In an explorative case study this would be the purpose of the case study as well as the criteria by which the case study will be judged successfully. It's essential that all these five steps are done prior to the collection of case study data. To make a prosperous case study analytic generalization rather than statistical generalization has to be used to derive conclusions. To have a good basis for the generalization it is vitally to have multiple sources of evidence, which might be documents, interviews, direct observation, participant observation or physical artefact. Ideally these different sources are analyzed separately, but the outcomes of their analysis is converging. Another fundament for the analytic process is a general analytic strategy, two suggested ones are to rely on theoretical propositions or to think about rival explanations for the collected information. The most common strategy to analyze the collected data, however, is pattern matching. This could mean to define separate variables and observe their development or to compare how well the rival explanations can explain the collected information. In addition to all of this, Yin suggests to create a case study database. This is a library of all collected data separate from the case study report, without the interpretation included in the report. This "increases markedly the *reliability* of the entire case study" (Yin,2003, p. 102), as the conclusions can be proved or the same data could be used for further analysis.

2.5 Summary

This literature review has first identified reasons for the usefulness group activities in learning can provide according to the great Russian psychologist Vygotsky. After this general approach, the author has justified his decision to choose co-operative learning to design the JVM environment. The main motivation for co-operative learning was the limited framework in which the JVM is developed and tested in this experiment. In the next two subsections current literature has been analyzed to derive guidelines and principles for the design of the case study about the JVM. First literature about teaching and learning mathematics has been investigated. The three most important outcomes of this are the following. First, mathematical problems should be more realistic and meaningful. And Second, think mathematically is at least as much about constructing, describing and explaining as it is about calculating. And third, it is very important to use different representational media for describing mathematic problems. In the following subsection of this literature review aspects induced by the use of computers have been revealed and four important guidelines have been discovered. First, co-operative learning using a computer needs structure and guidance. Second, a deliberate strategy has to be used to help students to reflect on their solutions. Third, the technology used has always to be reliable and accessible. And fourth, the attributes belonging to eMedia and the ways in which eMedia can be more useful for the illustration of mathematical problems than traditional media. In the final subsection about case studies it has become clear that case studies need a lot of theoretical framework to succeed in creating reliable conclusions. Amongst others five principles for the design of case studies have been stated as well as the need for a general analytic strategy and a case study database.

3 Design and Implementation

As mentioned in the abstract, the idea of this paper is to facilitate online co-operation about mathematics. As this is very broad, only one tiny aspect is tackled in the conducted case study. This aspect is how a tool to facilitate mathematical notation on a computer is used and if it facilitates and/or fosters online co-operation. In order to do this four friends of the author have agreed to participate in a case study. They were given a mathematical task and should discuss it via email. The tool they were given was the Java WebMatrix (JVM), a Java applet to create textual representations of matrices and their operations on a computer. The JVM has been specially developed for this case study and more information about it can be found in Appendix B. The friends have been students from different German and Austrian universities and different courses. This section will first describe the design of the case study, and why this design seemed to be appropriate according to the literature review, section 2. After this an overview of what actually happened will be given. However, there is also a case study database available, if you want to analyze the collected data on your own. At the end a short evaluation discusses if the original idea to foster online co-operation has succeeded or not.

3.1 Design of the case study

There are several reasons why a case study was chosen for this paper. First, other research strategies seemed inappropriate, as it was impossible to control the outer circumstances of the participants to conduct an experiment and it was also not suitable to just interview or survey the participants about how they would use the JVM. This is on one line with the literature review about case studies 2.4 , where it is stated that case studies are a proper research strategy to analyze "How" research questions. As said already, the research question of this case study is how a tool (here : the JVM) to facilitate mathematical notation on a computer is used and if it facilitates and/or fosters online co-operation. The propositions are that the participants are representing mathematical notation on a computer and that they do this to co-operate. As the idea is to analyze co-operation, the unit of analysis is the interpersonal communication, which in this case consisted solely out of emails, as the participants are at different places. After the data collection rival explanations are sought to see if and how the collected data matches the propositions. The purpose of the case study is to see how the JVM is used and will be judged successfully if the co-operation between the participants seems to work well. In addition to the data from the case study which is represented in this paper a case study database is available with all raw data. To match another outcome from the literature review there are three intentional sources for data. First there are pre- and post-questionnaires for the participants. Second there are the emails they sent when they worked on the task and third the actual solution they produced. In the following subparts the selection of the task and other decisions made due to the literature review about mathematical, co-operative and computer learning are stated.

3.1.1 Decisions due to mathematical learning

As shown in the literature review, there is currently a trend to change the kind of problems posed in mathematics. Rather than pose simplified problems pose realistic problems. To overcome the problem of the greater complexity of such a realistic problem, a real life problem was chosen to give the task a meaning, as this was suggested in the literature review. The actual task was to evaluate a survey of the quality of teaching and its long term effects. The complete task can be found in appendix D. The students are also asked to represent their thoughts in different media, as the JVM offers in addition to the representation of the mathematical notation of matrices and their operations field for comments so that it is possible to write down additional information about why different steps are done. These two decisions should lead to constructing, describing and explaining, in other words thinking mathematically.

3.1.2 Decisions due to co-operative learning

In order to incorporate the revelations of the literature review the following has been done. First of all it was shown that there are benefits of working together. And due to the restricted time constraints of this paper co-operative learning has been chosen as theoretical concept, as co-operative learning is more suitable for limited and well defined task to acquire basic skills. This is a little bit a contradictory outcome to the need of realistic and complex problems the literature review about mathematical learning asked for. The solution which is attempted in the case study is to pose a complex task, but to give clearly defined steps to the participants what to do. These steps can be found in appendix E, guidance for the task. In addition to this guidance for the task itself, a mathematic student was asked to help others if there are problems. And last, the author of this paper and conductor of the case study can send emails to help and restructure the process if the co-operation doesn't work and/or the situation becomes to complex. And the fact that each student is aware that all his emails are visible to the conductor of the case study is intended to prevent the task of failing because of the theory of social loafing.

3.1.3 Decisions due to computer based learning

Again the need for structure and guidance is mentioned, similar to the review of co-operative learning, so this has already been implemented in the design of the case study. Moreover the students should discuss their suggestions, so that there should be plenty of feedback for each participant. As technology a Java Applet was used, the JVM, and emails. Both are very common techniques for working online and seemed to be reliable and useable at all times. More details about the JVM can be found in appendix B. The server for the JVM was supplied by Netsoc, the computer science society of Trinity College Dublin. Again, a very reliable and grown up service which should guarantee access to the materials at all time. The superiority of eMedia is used as the JVM allows students to display both mathematical notation and written information, and the changes transfer between them. And the JVM is a small stand alone tool, which should be easy to integrate, or in other words small-but-easy-to-extend.

3.2 Roadmap of the conducted case study

The JVM was written and four friends of the author of this paper agreed to co-operate via email to work on the task, which can be found in appendix D. The students are Germans and Austrians from different universities. One mathematic student was chosen to participate to prevent that the task might be too complex for the group. The task was given in a Webquest, however, the participants could just read the task and the guidance for the task, appendix E, if they wanted to. Emails from the conductor of the case study and author of this paper were sent to tell the participants that it is relatively unimportant if they solve the mathematical task or not. The important thing is to use emails for co-operation, and if the JVM helps in this case. All these emails can be found in the case study database. The time period for the task was two weeks, this was clear to all participants from the beginning. Before and after the time for the task emails including the questionnaires, appendix C have been sent to the students. The pre-questionnaires have partly been used to determine the mathematical knowledge of the students and maybe have increased the amount of help if the participants are unfamiliar with too many of the expressions.

3.3 Evaluation

Altogether 30 emails have been sent related to the conducted case study. Below a quantitative representation of how these are spread

- 3 pre-questionnaires
- 4 post-questionnaires
- 5 emails about the task
- 5 emails from the case study conductor
- 13 emails about virus problems, mail deletions and acceptance of participation

Unfortunately the five mails about the task are not really well spread, four of them are from one participant. At the beginning of this section it has been said that this case study will be judged successfully if the co-operation between the participants works well. This, however, did not really occur. Only one participant deliberately asked the others for help with a problem and deliberately answered to a mail of the student who has done most of the work. If you want to come to this conclusion on your own the five emails which have been about the task are available in appendix F. This outcome was probably predictable after the pre-questionnaires, if regarded with the knowledge from the literature review. The literature review indicated that group co-operative or collaborative working is something which has to be taught and/or experienced frequently. However, in the restricted time frame of this case study it seemed to be impossible to include proper training how to do this. But the indication of the literature review was going in the right direction as only the student who stated in the pre-questionnaire, appendix C, to have been trained in group work and uses it frequently tried to start co-operation in this case study. How serious this lack of group working experience affected the case study is shown in the post questionnaires, where the mathematical student stated that the fact that she didn't know the other participants stopped her from participating. This was a major set back for the whole case study, as the mathematical student had a key role to guide and explain in the case study.

Another issue which hampered the co-operation were technical issues. Both students who didn't participate apart from the questionnaires stated problems with the supplied technology. Both noted that they couldn't run the JVM as they didn't had the Java software and didn't want to install it over their modem connection. Although the author wants to mention that one of these two students had a connection to a university network and is frequently using it for his entertainment. So there probably was a way to participate, but it certainly also was a barrier, as well. And last the task seemed to be too complicated. This was certainly proved when the two students who produced the five emails about the task both stated in the post questionnaire, appendix C, that they invested two hours of work in it. The idea of the case study conductor was that each student would have to spend altogether an hour on the task. The complicated task however was chosen directly due to the literature review about mathematical learning, which said that problems should be realistic. Obviously the solution which was tried in this case study, to pose a complicated task, but to provide single steps how this task can be solved didn't work.

The author doesn't see one absolute valid explanation for this. Yet there are different explanations which could have lead to this outcome. First, the participants might not have been motivated enough, as they just did it as a favor to the author of this paper and not in the usual rigid university system. Second, the spread of the information (the task was on a website, they had to read emails, had to use the JVM on another website) might have lead to confusion. Third, the lack of group learning experience of some participants certainly affected the case study. Probably there are other rivals for the explanation, but these three are the most likely ones for the author. The first one is founded on the fact that two participants didn't participate, apart from pre- or post-questionnaire. However, as mentioned above the other two students both spent two hours, so they definitely have been motivated. To amplify this, one of these two even has been in exam time and still spent two hours. And all participants replied to the post-questionnaires, another sign that they tried to participate. The third explanation finally is based on the fact that the mathematical student didn't participate, and that apart from the frequently in group learning engaged participant nobody tried to co-operate. A tiny fact which is distinguishing itself to the author is that the more engaged students always signed their emails, the others didn't. This however seems to be a basic matter of politeness if people are trying to co-operate.

After all the second and third explanations seems to be more relevant to justify the overall outcome of the case study. Reasons for the second one are that there definitely has been confusion, which can be very clearly seen in the first email in appendix F. In this a student suggested a way of solution for a first task. This first task, which was on the same website as the actual task of this case study was not meant to be solved in this case study. This was clearly indicated in the emails. In addition to this the other participating student had problems to access the sources for the case study as she was using different computers and didn't have access to all her emails at all times. Therefore the spread of the source information definitely hampered this task, and is a suitable explanation for the collected data by the author. But, as mentioned, the third explanation has also its validity, as it, amongst others, stopped the mathematical student from participating. Alone this matter of fact is enough to show the importance of the third explanation, the lack of group learning experience.

A little bit disappointing in this case study was that none of the participants ever used the possibilities of the JVM to display operations on a matrix. What is meant by this can be seen in the example output in appendix B.

3.4 Summary

At the beginning this section has highlighted decisions which were made in the design of the case study about co-operation via email about mathematics. The foundations for these decisions were given in the literature review and were discussed separately to their origin. Decisions were made due to learning about mathematics, co-operative learning and computer based learning, and of course the design of case studies. After this a roadmap described the main sequence the case study followed, and revealed that the participants in this case study have all been friends to the author, which is probably important to know. A central decision due the literature review about case studies was to offer a case study database, which should be available at the same website where you have retrieved this paper. At the end this section has roughly evaluated the collected data. The proposition of the case study to foster mathematical co-operation has not become true. However, the JVM has been used to display mathematical notation on a computer. Possible reasons for this are technical problems and lack of experience to work in groups. Summarized, the two most relevant explanations to describe the outcome seem to be the spread of source information and the lack of group learning experience. The following conclusion highlights the main facts which have been discovered during this case study and gives suggestions if and what further work may be needed.

4 Conclusion

As evaluated in the previous section the case study has not really succeeded to foster asynchronous co-operative learning via email in mathematics. Hence the case study has not been a complete success, but it has given fingertips that the subject of the case study to foster mathematical co-operation via a tool to display mathematical notation might be helpful. This is because all participants stated that the JVM might be of use for this in the post-questionnaire, so there seems to be a reason to conduct further case studies with more time and hence improved possibilities to foster online co-operation in mathematics. Possible enhancements needed can be derived from facts which have been discovered in this case study that probably have hampered the creation of co-operative learning. Two are crucial. First, participants in a similar case study should definitely be already or become trained to work in groups. The other significant fact is that the spread of source information has definitely hampered this case study. Additional changes might be to use a regular university assignment as frame for a case study, so that the possibility that some participants drop out is less. Or the task could be facilitated, although the literature about learning mathematics indicates differently.

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A Emails from lerngruppe

Below some excerpts from the email group which lead to the idea of this paper. Names are replaced with xxx, sorry they are only available in German.

```
Date: 27/06/02 17:17

Hi allerseits!

Falls es jemanden interessieren sollte, kommt hier die Loesung der
4.Aufgabe:


der Ergebnisvektor lautet: (0,1,1,2,1,1,0,0) und das ganze
natuerlich transponiert, irgendwie habe ich keine grosse Lust die
DFT8 und die Inverse mitzuschicken, weil das zu lange dauert, hab
ich aber morgen dabei.

Gruesse xxx

PS: das Ergebnis stimmt, denke ich, weil ich es mit jemand anderem
verglichen habe, aber ueber Einwaende freue ich mich trotzdem

-------------------------------------------------------

Date: 21/07/02 22:57


CIAO!

Dank erstmal fuer deine Hilfe...aber nochmal klaerungen werden
verlangt :) :

> 2. In der LR-Zerlegung mit Spaltenpivotsuche; welche Zeilen muss man denn
> nun vertauschen? So richtig hab ich das nun nicht geblickt? Was muss nun
> groesser sein als was??? Und ganz so nebenbei: kann man denn von P A = L R
> ohne weiteres kommen? D.h., wenn ich PA gegebon hab, kann ich das einfach so
> auf LR rueberzaubern, oder tut er das im "Skript" nur so nebenbei um nicht
> kostspieliges Papier zu verbrauchen???

> ... Du tauschst soviel ich wei die Zeile in der Du dein Pivotteil
>   gefunden hast nach oben!

Na, und was ist denn mein Pivotschrott? Da steht was von
"vertausche im k-ten schritt die k-te zeile..." irgendwas
```

```
maximales muss kleiner irgendwas sein... kann das jemand auch auf
umgangsprache? zB dachte ich (zumindest hab ichs so verstanden):
das groesste element einer zeile darf nicht groesser sein als ein
bestimmtes (WELCHES?!?!) einer (oder von ALLON?!?!) zeile(n)
davor, sonst muss man tauschen. Aber an dem gegebenen beispiel
geht meine theorie zugrunde...

--------------------------------------------------------

Hallo,
 in der Aufgabe werden durch Gau'sche Umformungen drei Polynome (p,q,r) zu einer
 Basis von  [x] ergaenzt. Klingt einfach. Die Koeffizienten der Polynome in die
  Zeilen einer Matrix schreiben - Umformen - und dann ablesen.
 Loesungsmenge ist dann:
 {p,q,r}  {x^i | i   \ {3,4,6} }
 3,4,6 sind in diesem Beispiel die Spalten, in denen Stufen entstanden sind.
 So wurde es in der bung gemacht. Was ich aber nicht verstanden habe... In
 der 2. Spalte (also fuer x^1) wurden in jeder Zeile Werte stehen gelassen.
 Muesste man diese nicht ebenfalls ausraeumen?

 Matrix:
 0 7/3 0 1 0 0 0
 0 -3 0 0 1 0 0
 0 -9 0 0 0 0 1
```

B Java WebMatrix

The JVM is a Java Applet and is intended to run on a webserver, during this project the webserver will be provided by Netsoc, the computer science society a Trinity College Dublin. There are two major reasons for the decision to use an Java Applet. First, the author believes that it is a barrier if users first have to install a piece of software on their own machine before they can run it. Second, the JVM should be easy accessible for as much users as possible. A Java Applet offers a solution to both problems as nearly all computers have a webbrowser installed and you don't have to install software to run it.

The interface of the artefact is as easy to use as possible. There are comments field between the matrices to include explanations. A restriction to facilitate both the programming part as also the user interface is that the JVM will offer only one matrix operation at each step, the meaning of this shown below

$$
Allowed \begin{pmatrix} 1 & 1 & 1 \\ t & 2t & 2 \\ t+1 & 0 & 2t \end{pmatrix} \begin{array}{l} -t \\ + \\ \end{array}
$$

$$
NotAllowed \begin{pmatrix} 1 & 1 & 1 \\ t & 2t & 2 \\ t+1 & 0 & 2t \end{pmatrix} \begin{array}{ll} -t & -(t+1) \\ + & \\ & + \end{array}
$$

This facilitates both the programming part, as also the understanding of the created files. However, experienced users probably don't like it, but the aimed focus group for JVM are people who are new to matrices, therefore it is a bearable restriction. An example of the JVM output is shown below

```
JavaWebMatrix Output

    (   1   2    3  )  --|
    (   e^x 45   4  )    |
    (   -9  53   6  )  <-|  x (9)

Multiply row 1 with 9 and add to row 3

    (   1   2   3   )
    (   e^x 45  4   )  <-|
    (   0   71  33  )  <-|


Swap row 2 with row 3

    (   1   2   3   )
    (   0   71  33  )    |  x (4)
    (   e^x 45  4   )


Multiply row 2 with 4

    (   1   2   3   )
    (   0   284 132 )
    (   e^x 45  4   )
```

C Questionnaires

C.1 Pre-Questionnaire

1. How many of the following terms you know and have used ?

- basic matrix operations
- gaussian elimination of matrices
- linear independent vectors
- orthogonal vectors
- diagonalisation of a matrix
- eigenvalue
- determinant of a matrix

2. Have you worked in the last 12 months with matrices?
3. Have you ever had problems with a task including matrices and found out that you stumbled over a silly/little/stupid obstacle?
4. Do you know a way to display matrices and their operations on a computer?
5. Do you often work in group projects?
6. Have you ever been taught how to successfully perform in group work?
7. Have you ever co-operated with people via computers over a distance?
8. If you answered yes to question 7, did you encounter problems? Which?
9. What do you expect from this project? (1-3 sentences)

C.2 Post-Questionnaire

1. Did you send any email for this task?
2. If you answered question 1 with no, why not? Which problems? What did you not like?
3. Did you try the WebMatrix Applet? You can answer yes, even if you haven't send an email.
4. Did you encounter problems by using the Applet?
5. Have you been daunted to participate because you didn't know the others?
6. Do you think that my Java Applet helps if you want to ask a question about a matrix in an email?
7. Do you have any suggestions what should have been different in this task?
8. How much time did you spend for this task (including everything)?

D The task

In a land far away, Ellatien recently the following has occurred. The ministry for waste economy, mathematic and industry (WEMI) has been flushed by the LISA-survey, which was judging the quality of teaching. Immediately starting with activism, WEMI asked the popular Prof. Dr. No-Way, to pursue a survey about the attendance of lectures at the university. Surveyed was the lecture in background studies, respectively the subject Ella. Prof. Dr. No-Way finished after a nearly representative survey to the following conclusion. Not all of the 880 signed in students always go to the lectures ! More detailed he came up with :

- Out of 10 students, who are going on any day in the Ella lectures, only 7 students return the next time. 2 are going in a pub and one is going into another lecture by mistake.
- Out of 10 students, who are going on any day into another lecture by mistake, one is doing it the next time again. 4 prefer to going to the pub, 3 stay in bed and 2 are going in the Ella lectures.
- Out of 10 students, who prefer going to a pub instead of going to the Ella lectures, 4 are repeating this the next time again. 4 instead stay in bed, 1 is going to the correct lecture and 1 is going to the wrong lecture.
- Out of 10 students, who prefer to stay in bed on any day, 7 will repeat this the next time. The 3 other people distribute equally to pub, correct and wrong lecture.

WEMI can make nothing of this. Help WEMI !
Calculate how many students will be in the correct lecture, pub, bed and wrong lecture at the end of the lecture time (15 weeks with 2 lectures)! Show also, that the result doesnt change, if already at the start of the semester only half of the students is going in the correct lecture.

E Guidance for the task

E.1 Definition of variables

x_1: number of students in the correct lecture
x_2: number of students in wrong lecture
x_3: number of students in a pub
x_4: number of students in bed
$\vec{x} = (x_1, x_2, x_3, x_4)$, the current student situation vector
standard basis vectors $\vec{e}_1, \vec{e}_2, \vec{e}_3$ and $\vec{e}_4$
A is the transformation matrix
phi represents the function which is multiplying a vector with A
the Eigenvectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ and $\vec{b}_4$

D is the diagonalized version of A
B is a basis of Eigenvectors
$\vec{y} = (y_1, y_2, y_3, y_4)$, the current student situation vector affected to

E.2 Roadmap for the task

- define A and phi
- determine the Eigenvalues of A. To do this you need to know how to calculate a Determinant.
- a little help here, as you found four Eigenvalues, the matrix is diagonalizable. The diagonalized matrix D is a matrix with only zeros except the diagonal which consists of the four Eigenvalues.
- D is the transformation matrix of phi affected to any basis of Eigenvectors. For our purpose it is enough if you just know that this basis is a matrix with the four Eigenvectors as its columns. (the basis we have worked on before was the standard basis, which consisted out of the four standard basis vectors and consists of zeros except the diagonal which has four one's on it).
- Now we have to solve By = x to gain the number of students at the beginning affected to B. (If we are doing this we can use D to calculate phi, instead of using A, which would be much more complex)
- Finally D^ny gives us the current situation after n lectures.
- Still to answer the additional part of the task, how it would change if the beginning situation would change, but if you made it till here this should be possible for you now

F Excerpts of collected data

Following the mails about mathematical issues which have been sent due to this task. Only to students have done this, the name of the one student is replaced with xxx, whereas the name of the other student is replaced with yyy. As you can see the main part of work has been done by the single student represented through xxx.

```
Hi,

Task 1:

I tried to solve the problem with gaussian elimination of matrices
(see attachment). Unfortunately it did not work. The funtion,
which models the shoveling, does not increase linear (?).
```

Any ideas?

xxx

JavaWebMatrix Output

```
    (   2   25/3    )
    (   5   6       )
    (   11  a       )
```

Hi,

Task 2:

I think I got the Matrix T (see attachment) we need to solve the problem. When you multiply the vector containing students going to "(lecture, pub, other lecture, bed) transformed" from the left side of the matrix, you get the vector of students for the next time.

I think, we need to multipy the matrix with the startvector 30 times, then we will get the solution.

Maybe it would be easier to find the invariants of the matrix, then we would not need that much multipliing.

What do you think?

xxx

JavaWebMatrix Output

```
    (   7/10    2/10    1/10    1/10    )
    (   2/10    4/10    4/10    1/10    )
    (   1/10    1/10    1/10    1/10    )
    (   0       3/10    4/10    7/10    )
```

Sorry must be multipliing from the right side.

Servas guys!

Okido, here comes the 'business input' on our little mathematical issue...

Anyway, I had to read the process section first before I even got to the point of understanding the task. I knew what was meant by "linear system of equations", I remember that from school:-) but the matrices confused me a little bit... I agree though with the proposal of xxx. Basically, I got the part with choosing variables for each group of people and also putting them into relation to each other but the part with 'phi' and Eigenvectors... nooooo way! And what the hell are matrix A or matrix T?

Greetings from Dublin,

yyy

P.S.: Can we invite Keanu Reeves to our little discussion group?!

Hi,

I think the matix is ugly. Getting the eigenvalues is nasty work. Unfortunatelly I uninstalled maple. So here is my theoretical solution:

Take Matrix A:

```
( 7/10     2/10     1/10     1/10 )
( 2/10     4/10     4/10     1/10 )
( 1/10     1/10     1/10     1/10 )
( 0        3/10     4/10     7/10 )
```

diagonalize it (determinant of A-x) = diag(A) .

Get Matrices T and T^-1 from:

A = T^-1 * diag(A) * T

So we easy get:

A^30 = T^-1 * diag(A)^30 * T

And the solution is:

T^-1 * diag(A)^30 * T * s   s is startvector

What do you think? Anybody installed maple/mathematica or something like that?

xxx