# Solutions to the WebQuest

Martin Raab

February 23, 2004

## 1 Aunt Hilda Problem

Definition of variables :

- **m**: the amount of snow which is falling per day on the area of the university
- **t**: the amount of days the teams are waiting before they start with shovelling snow
- **a**: amount of snow the first team (2 persons) shovelled
- **b**: amount of snow the second team shovelled (5 persons)
- **c**: amount of snow the third team shovelled (11 persons)
- **p** amount of snow one person shovels on one day
- **x**: number of days the third team needs to shovel the university snow free

Hence m*t is the amount of snow which has fallen before the teams start to shovel

Team A shovels 1/4 of the university snow free, which means they shovelled 1/4 of the amount of snow which has fallen before they started (m*t) and 1/4 of the amount of snow which has fallen during the shovelled, which is 8*m, because they shovelled 8 days.

expressed in a formula this is :
a = 1/4*(m*t) + 1/4*(8*m) = 1/4*(m*t + 8*m)

similar we gain for the other teams the formulas

b = 1/2*(m*t + 6*m) and c = (m*t + x*m)

According to the hint we now try to calculate the amount of snow one person shovels per day. This is the total amount of snow one team shovels divided by the number of days and the number of persons.

p = a / (2*8) = a / 16
p = b / (5*6) = b / 30
p = c / (11*x)

If we now equate the upper two equations and insert the expressions we have for a and b we have the following

(1/4(mt+8m))/16 = (1/2(mt+6m))/30
1/64(mt+8m) = 1/60(mt+6m)

now we have to multiply with the smallest common denominator, which is 15*16*4 = 960 and results in

15(mt+8m)=16(mt+6m)
mt = 24m
t = 24

Voila, we have our first variable determined. Now we use the lower two equations and do the same again

(1/2(mt+6m))/30 = (mt + xm)/11x

and insert the value of t, 24

1/60(24m+6m) = (24m + xm)/11x
m/2 = m*((24+x)/11x)

divide through m

1/2 = 24+x/11x
11x/2 = 24 + x
11x = 48 + 2x
x = 5 1/3

And we are finished the needed 5 and 1/3 days to shovel the hole university snow free. ( assumed that they shovelled all day and night :)

# 2 WEMI Task

## 2.1 Definition of variables

$x_1$: number of students in the correct lecture
$x_2$: number of students in wrong lecture
$x_3$: number of students in a pub
$x_4$: number of students in bed
$\vec{x} = (x_1, x_2, x_3, x_4)$, the current student situation vector
standard basis vectors $\vec{e}_1, \vec{e}_2, \vec{e}_3$ and $\vec{e}_4$
$A$ is the transformation matrix
*phi* represents the function which is multiplicating a vector with A
the Eigenvectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ and $\vec{b}_4$
$D$ is the diagonalized version of A
$B$ is a basis of Eigenvectors
$\vec{y} = (y_1, y_2, y_3, y_4)$, the current student situation vector affected to B

## 2.2 Notes

Just to mention here, this solution won't show every step of calculation, as this would take ages and is quite hard to do on a computer. Instead it will show the main intermediary results and the way of calculation (As it was in the computer science course in Germany).

As this solution is not written for mathematic experts, I'll try to skip as much special terms as possible. This definitely increases the amount of text needed to describe this solution, and in some cases it might even become slightly unprecise. But you should notice that this solution could be much briefer if you would use words as basis transformation, linear dependent, vector space ....

## 2.3 Transformation Matrix A and function phi

First we regard the standard system of linear equations most people would usually use. To do this, we assume that at the beginning all people do one certain activity, in the vector language this would be one standard basis vector from $\vec{e}_1..\vec{e}_4$. After this definition we can define that $\vec{e}_1$ represents the state that all students go to the correct lecture.

$phi(\vec{e}_1) = 0.7x_1 + 0.1x_2 + 0.2x_3$
$phi(\vec{e}_2) = 0.2x_1 + 0.1x_2 + 0.4x_3 + 0.3x_4$
$phi(\vec{e}_3) = 0.1x_1 + 0.1x_2 + 0.4x_3 + 0.4x_4$

$phi(\vec{e}_4) = 0.1x_1 + 0.1x_2 + 0.1x_3 + 0.7x_4$
The above in words : as said $\vec{e}_1$ means that 100% of all students go to the lecture. The next time, and this is what phi() should calculate 70% of the students are in the correct lecture, 10 % in the wrong lecture and so on. Now, as described on the process page, we basically just leave away all the variables, and have our transformation matrix A (and hence phi, too).

$$\mathbf{A} = \frac{1}{10} * \begin{pmatrix} 7 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 1 \\ 0 & 3 & 4 & 1 \end{pmatrix}$$

### 2.4 Eigenvalues and Eigenvectors

If we now have $\vec{x}$, which represents the amounts of students on a certain day, then $A * \vec{x}$ represents the amounts of students on the next day.

After calculation we figure out that 0, 0.3, 0.6 and 1 are the Eigenvalues of A. With the help on the process page we now know that A is similar to D, which is represented below.

$$\mathbf{D} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The next step is to calculate the Eigenvectors. They are shown below.

$$\vec{b}_1 = \begin{pmatrix} 3 \\ -18 \\ 17 \\ -2 \end{pmatrix} \vec{b}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} \vec{b}_3 = \begin{pmatrix} 3 \\ 0 \\ 4 \\ 1 \end{pmatrix} \vec{b}_4 = \begin{pmatrix} 11 \\ 4 \\ 9 \\ 16 \end{pmatrix}$$

Thus B is the following :

$$\mathbf{B} = \begin{pmatrix} 3 & 0 & 3 & 11 \\ -18 & 0 & 0 & 4 \\ 17 & 1 & 4 & 9 \\ -2 & -1 & 1 & 16 \end{pmatrix}$$

### 2.5 Basis Transformation, or By = x

The student numbers can generally be described as $\vec{x} = (a \quad b \quad c \quad 880 - a - b - c)$. Which means, In bed are exactly the students whoa are nowhere else. As said, we want to transform this into B by solving By = x.

$$\left(\begin{array}{cccc|c} 3 & 0 & 3 & 11 & a \\ -18 & 0 & 0 & 4 & b \\ 17 & 1 & 4 & 9 & c \\ -2 & -1 & 1 & 16 & 880 - a - b - c \end{array}\right)$$

which leads to the following

$$\left(\begin{array}{cccc|c} 3 & 0 & 3 & 11 & a \\ -18 & 0 & 0 & 4 & b \\ 17 & 1 & 4 & 9 & c \\ 0 & 0 & 0 & 1 & 22 \end{array}\right)$$

Before we continue solving this matrix problem, we think about what we want to do. We want to calculate the numbers of students after n lectures, which basically means $D^n * \vec{y}$. But $D^n$ is becoming more and more like shown below

$$\mathbf{D}^n = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This means that all we need is $y_4$, which can be read out as 22 from the last line in the changed version of By = x. We now have to retransfer y back into the standard basis, for those who don't know this means multiplying $b_4$ with 22. And this is the result.

242 students are going in the correct lecture
88 students are going in the wrong lecture
198 students are going in the pub
358 students are staying in bed

As we can see the final numbers of students doesn't depend on the starting values a, b and c. Therefore we have answered the additional question as well, if we only chose n big enough.